THE EFFECTS OF FARADAY ROTATION ON BACKSCATTER SIGNATURES IN SAR IMAGE DATA

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Abstract

The effect of Faraday rotation on spaceborne polarimetric SAR measurements is addressed. Single-polarized, dual-polarized and quad-polarized backscatter measurements subject to Faraday rotation are modeled. It is shown that due to Faraday rotation, the received signal includes other polarization characteristics of the surface. Techniques are developed to detect the presence of Faraday rotation in dual-polarized and quad-polarized SAR data. Finally, a novel approach for the correction (or calibration) of linearly polarized fully polarimetric data for Faraday rotation, to recover the 'true' scattering matrix, is presented.

1. IONOSPHERIC EFFECTS

The ionosphere is defined to be the region of the upper atmosphere with large quantities of charged particles (an ionized medium). This ionized medium becomes anisotropic in the presence of a steady magnetic field such as the Earth's magnetic field. Radio waves propagating through the ionosphere therefore experience a rotation of polarization vector known as Faraday's rotation. The magnitude of the Faraday rotation angle depends on the frequency of the wave and the direction of the earth's magnetic field and its plasma electron density. Since the parameters of the earth's ionosphere are dynamic and their fluctuations depend on diurnal, seasonal, latitudinal and solar cycle effects, the accurate calculation of the Faraday rotation of the polarization vectors is difficult. Therefore, we use the nominal values of electron density at the frequency of operation and the magnitude of the earth's magnetic field assuming a vertically homogeneous ionosphere model.

In general the Faraday rotation angle of a linearly polarized wave integrated over the path length is half the phase difference between the right and left circularly polarized waves. The total Faraday rotation angle is given by:

$$\Omega = \frac{e^3\lambda^2}{8\pi^2 c^3 \varepsilon_0 m^2} \int_{path} NB \cos\theta dr \qquad (1)$$

where
e= charge of an electron (1.602 x 10^{-19} coolomb)
m= mass of an electron (9.1 x 10^{-31} kg)
c= velocity of light (3x 10^{8} m)

ε_0 = permittivity of vacuum (8.85 x 10^{12} farad m^{-1})

λ = radio propagation wavelength, m

N= number of electrons,

B= magnetic flux density, Webers m^{-1}

θ = angle between B and direction of wave propagation

The variables B, N, and e depend on several dynamic parameters of the ionosphere as mentioned earlier. The distribution of these parameters along the propagation path are difficult to determine. Maximum electron densities are found in temperate latitudes and in winter during midday. In regions near the geographic equator the electron density may be smaller. From the literature, the electron density also varies with respect to the height, reaching its maximum at between 300 to 400 km within the F2 region of the ionosphere. By assuming constant electron density, Earth's magnetic field and its angle along the propagation path, equation (1) can be reduced to:

$$\Omega = 2.631\mathrm{X}\,10^{-13}\lambda^2 BNR\cos\theta \qquad (2)$$

where R is the path length in meters and N is the electron density at the spacecraft height. The maximum value of the electron density for different latitudes can be obtained by employing the critical frequency of the ionosphere (maximum plasma frequency) in the following expression:

$$N = 1.24 f_c^2 \times 10^{10} \qquad (3)$$

The critical frequency can be derived from measurements and ionization maps that are often made for each hour of each month and for both solar minimum and maximum. Equation (3) is employed to derive the limiting cases for electron density of the ionosphere. Note that in equation (2), we assumed the electron density is constant along the path. Therefore, the limiting cases are used to calculate the range of Faraday rotation of the received signal for a satellite at 568 x10^3 m altitude and the observation at about midday. The angle between the Earth magnetic field and the direction of propagation can be approximated as constant at θ = 51.50= (90.0 -38.50).

Table 1 provides the limiting cases of the Faraday rotation angle in degrees for regions at about ± 15" from the equator. The calculations are performed for both midday and midnight. The table shows that Faraday rotation may be significant for L-Band and P-Band spaceborne SARS, but not for C-Band.

Table 1. Ranges of critical frequency, electron density and the Faraday rotation angle for a radar in a 568 km, polar orbit, with a 38.5 degree look angle over the tropical region (i- 15“ from the equator). Faraday rotation angles are given for C-Band (6 cm wavelength), L-Band (24 cm wavelength), and P-Band (68 cm wavelength).

	Midnight		Midday	
	Min	Max	Min	Max
$f_c(MC/S)$	5	9	11	12
N(electrons/m^3)	0.31 X10$^{'2}$	1.004 X10$^{'2}$	1.5 x 10$^{'2}$	2.79 x10$^{'2}$
Ω_C (degrees)	0.3°	0.9°	1.4°	2.6°
Ω_L (degrees)	4.6°	14.8°	22.05°	41.00
Ω_P (degrees)	36.61°	118.6°	177.0°	329.5°

As a result of propagation through the ionosphere, Faraday rotation affects the SAR system both in azimuth and range direction in a complex fashion. In range direction, the effect can be compensated during the range compression because the antenna is fixed with respect to the scatterers in a given range pulse. In the azimuth direction, the antenna moves relative to the scatterers and the Faraday rotation factors are convolved with the scattered wave from the scene (Rosen, 1992; Gail, 1995). In what follows, we will consider only the net Faraday rotation in the final image product, i.e. the result after both azimuth and range compression. We will also simplify the situation, in that Faraday rotation is the only source of system calibration error considered here.

Faraday rotation is a significant obstacle to the deployment of longer wavelength spaceborne SARS for geophysical applications. Unless a solution can be found to this problem, backscatter measurements made by such radars will be difficult if not impossible to calibrate, which will mean that they will not be useful in most geophysical algorithms which use radar measurements, for example in calculating biomass or soil moisture. In this paper, we discuss how the presence of Faraday rotation in SAR image data may be identified, and we introduce a new approach to correct for it in the case of fully polarimetric measurements made by a linearly polarized transmit) receive radar system.

20 MODELING THE EFFECT ON BACKSCATTER SIGNATURES

By assuming that there is no variation of Faraday rotation across the SAR antenna beam, linearly polarized transmitted and received waves by the SAR system can experience two incidence of Faraday rotation, one downgoing and onc upgoing. The sense of Faraday rotation in each direction is the same relative to the Earth's magnetic field and is independent of the propagation direction. For a general case, let's assume the transmitted electric field, **Et can be** represented in terms of horizontal and vertical field components. After one way transmission through the ionosphere, the field incident, **B,** on the Earth's surface is rotated by the Faraday rotation matrix **R** such that $\mathbf{E_i = RE_t}$. The scattered field from the target (distributed target in case of land surface) undergoes another rotation along the receiving propagation path before reaching the antenna. Therefore, the elements of the scattering matrix, S, of the target are modified as a result of the round-trip Faraday rotation angle. As described in (Gail, 1995), for a SAR system measuring linear horizontal (H) and vertical (V) polarizations in the antenna coordinate system, the measured scattering matrix, **M,** can be written as **M = RSR, i.e.**

$$\begin{bmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} S_{hh} & S_{vh} \\ S_{hv} & S_{vv} \end{bmatrix} \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \tag{4}$$

which can be written:

$$M_{hh} = S_{hh}\cos^2\Omega - S_{vv}\sin^2\Omega + \left(S_{hv} - S_{vh}\right)\sin\Omega\cos\Omega \tag{5a}$$

$$M_{vh} = S_{vh}\cos^2\Omega + S_{hv}\sin^2\Omega + \left(S_{hh} + S_{vv}\right)\sin\Omega\cos\Omega \tag{5b}$$

$$M_{hv} = S_{hv}\cos^2\Omega + S_{vh}\sin^2\Omega - \left(S_{hh} + S_{vv}\right)\sin\Omega\cos\Omega \tag{5c}$$

$$M_{vv} = S_{vv}\cos^2\Omega - S_{hh}\sin^2\Omega + \left(S_{hv} - S_{vh}\right)\sin\Omega\cos\Omega \tag{5d}$$

Invoking backscatter reciprocity, i.e. $S_{hv} = S_{vh}$,

$$M_{hh} = S_{hh}\cos^2\Omega - S_{vv}\sin^2\Omega \tag{6a}$$

$$M_{vh} = S_{hv} + \left(S_{hh} + S_{vv}\right)\sin\Omega\cos\Omega \tag{6b}$$

$$M_{hv} = S_{hv} - \left(S_{hh} + S_{vv}\right)\sin\Omega\cos\Omega \tag{6c}$$

$$M_{vv} = S_{vv}\cos^2\Omega - S_{hh}\sin^2\Omega \tag{6d}$$

Note that for cross-pol measurements, the presence of non-zero Faraday rotation means that they will not necessarily be reciprocal, i.e. $M_{hv} \neq M_{vh}$.

2.1 HH polarization measurements only

Several spaceborne SARS have been flown which measure L-Band HH polarization backscatter only, i.e. Seasat SAR, SIR-A, SIR-B and JERS- 1. The expected value of the radar cross section measured by these radars in the presence of Faraday rotation is:

$$\left\langle M_{hh}M_{hh}^{*}\right\rangle = S_{hh}S_{hh}^{*}\cos^4\Omega - 2\mathrm{Re}\left(S_{hh}S_{vv}^{*}\right)\sin^2\Omega\cos^2\Omega + S_{vv}S_{vv}^{*}\sin^4\Omega \tag{7}$$

We have also modeled the effect of Faraday rotation on Repeat-pass Interferometry measurements made by HH-polarization-only spaceborne radars. This is done by correlating a 'nominal' measurement of S_{hh} with zero rotation with a subsequent rotated measurement as given in (6), to give:

$$\left\langle M_{hh_1} M^*_{hh_2} \right\rangle = S_{hh_1} S^*_{hh_2} \cos^2\Omega - S_{hh_1} S^*_{vv_2} \sin^2\Omega \tag{8}$$

Assuming no change in the backscatter, i.e. that there is no temporal decorrelation, we can assess the decorrelation introduced by Faraday rotation alone:

$$\left\langle M_{hh_1} M^*_{hh_2} \right\rangle = S_{hh} S^*_{hh} \cos^2\Omega - S_{hh} S^*_{vv} \sin^2\Omega \tag{9}$$

As a normalized correlation coefficient, this can be expressed as:

$$\rho_{\text{Faraday}} = \frac{\left| \left\langle M_{hh_1} M^*_{hh_2} \right\rangle \right|}{\sqrt{\left\langle M_{hh_1} M^*_{hh_1} \right\rangle \left\langle M_{hh_2} M^*_{hh_2} \right\rangle}}$$

or

$$\rho_{\text{Faraday}} = \frac{\left| \left(S_{hh} S^*_{hh} \cos^2\Omega - S_{hh} S^*_{vv} \sin^2\Omega \right) \right|}{\sqrt{\left(S_{hh} S^*_{hh} \right) \left(S_{hh} S^*_{hh} \cos^4\Omega - 2\mathrm{Re}\left(S_{hh} S^*_{vv} \right) \sin^2\Omega \cos^2\Omega + S_{vv} S^*_{vv} \sin^4\Omega \right)}} \tag{10}$$

which does not include scene-dependent temporal decorrelation or noise decorrelation.

2.2 Dual-polarized (HH and HV) measurements

The SIR-C radar instrument had one mode in which only HH and HV measurements were made at a given frequency. This allowed for wider swaths to be illuminated than fully polarimetric modes because of the lower data rates. This type of data collection has also been proposed for other planned spaceborne SARS such as LightSAR and PALSAR. Forming cross-products between the scattering matrix terms in (6), wc obtain, in the presence of Faraday rotation:

$$\left\langle M_{hh} M^*_{hh} \right\rangle = S_{hh} S^*_{hh} \cos^4\Omega - 2\mathrm{Re}\left\{ S_{hh} S^*_{vv} \right\} \sin^2\Omega \cos^2\Omega + S_{vv} S^*_{vv} \sin^4\Omega$$

$$\left\langle M_{hv} M^*_{hv} \right\rangle = S_{hv} S^*_{hv} + \sin^2\Omega \cos^2\Omega \left(S_{hh} S^*_{hh} + S_{vv} S^*_{vv} + 2\mathrm{Re}\left\{ S_{hh} S^*_{vv} \right\} \right)$$

$$\left\langle M_{hh}M_{hv}^{*}\right\rangle \quad \sin\Omega\cos\Omega\left(-S_{hh}S_{hh}^{*}\cos^{2}\Omega - S_{hh}S_{vv}^{*}\cos^{2}\Omega - S_{vv}S_{hh}^{*}\sin^{2}\Omega + S_{vv}S_{vv}^{*}\sin^{2}\Omega\right) \tag{11}$$

where we have assumed that azimuthal symmetry holds, as is the case for most natural targets, with the consequence that:

$$\left\langle S_{hh}S_{hv}^{*}\right\rangle = \left\langle S_{hv}S_{vv}^{*}\right\rangle = 0$$

2.3 **Quad-polarized measurements**

SIR-C also made fully polarimetric or quad-pol backscatter measurements. Such measurements are also planned for the LightSAR L-Band radar system. During the calibration of SIR-C polarimetric data, a symmetrization operation was performed. In the process of symmetrization, the cross-product between the two like-pol measurements is used to assess any amplitude or phase imbalance between these two channels. This cross-product, from (6b) and (6.), would be:

$$\left\langle M_{hv}M_{vh}^{*}\right\rangle = S_{hv}S_{hv}^{*} - \left(S_{hh}S_{hh}^{*} + S_{vv}S_{vv}^{*} + 2\mathrm{Re}\left\{S_{vv}S_{vv}^{*}\right\}\right)\sin^{2}\Omega\cos^{2}\Omega \tag{12}$$

If the phase amplitude of this cross-product exceeded certain thresholds when compared with default system values, the default values were used in symmetrization. The net effect would be to add the HV and VH measurements after correction for all system-dependent phase and amplitude imbalances, i.e. the composite 'symmetrized' HV measurement would be obtained from:

$$M_{hv}^{'} = 0.5\ \left(M_{hv} + M_{vh}\right) \equiv S_{hv} \tag{13}$$

Thus for properly 'symmetrized' cross-pol data, the Faraday rotation should have no effect on the measured backscatter value. Forming cross-products from (6a), (6d) and (12), we obtain:

$$\begin{aligned}
\left\langle M_{hh}M_{hh}^{*}\right\rangle &= S_{hh}S_{hh}^{*}\cos^{4}\Omega - 2\mathrm{Re}\left\{S_{hh}S_{vv}^{*}\right\}\sin^{2}\Omega\cos^{2}\Omega + S_{vv}S_{vv}^{*}\sin^{4}\Omega \\
\left\langle M_{hv}M_{hv}^{*}\right\rangle &= S_{hv}S_{hv}^{*} \\
\left\langle M_{vv}M_{vv}^{*}\right\rangle &= S_{hh}S_{hh}^{*}\sin^{4}\Omega - 2\mathrm{Re}\left\{S_{hh}S_{vv}^{*}\right\}\sin^{2}\Omega\cos^{2}\Omega + S_{vv}S_{vv}^{*}\cos^{4}\Omega \\
\left\langle M_{hh}M_{vv}^{*}\right\rangle &= S_{hh}S_{vv}^{*}\cos^{4}\Omega - \left(S_{hh}S_{hh}^{*} + S_{vv}S_{vv}^{*}\right)\sin^{2}\Omega\cos^{2}\Omega + S_{vv}S_{hh}^{*}\sin^{4}\Omega \\
\left\langle M_{hh}M_{hv}^{*}\right\rangle &= 0 \\
\left\langle M_{hv}M_{vv}^{*}\right\rangle &= 0
\end{aligned} \tag{14}$$

where again only the like-pol terms are affected by the Faraday rotation. A corollary "of" this is that, if the default 'symmetrization' is not successful, the effects would quickly be evident as non-zero $M_{hh}M_{hv}*$ and $M_{hv}M_{vv}*$ terms.

3. SAMPLE RESULTS

To determine what will happen to backscatter measurements affected by Faraday rotation, it is clear that the full scattering matrix for the backscatter should be known. The polarimetric backscatter measurements in Table 2 were extracted from L-Band AIRSAR data obtained over a tropical rain forest in Belize during 1991.

Table 2: L-Band backscatter measurements from AIRSAR data obtained over Belize, in April 1991

L-Band	HH	HV	VV	HH-VV Phase	HH-VV Correlation
Bare Soil	-16.5	-269	-14.7	-23.7	0.75
Farmland	133	-25	-11.8	-18.6	0.75
Upland Forest	-9.2	-14.3	-9.4	7.9	0.25
Swamp Forest	-6.9	-14.5	-7.3	165.4	0.06
coffee	8	-15.7	-9.7	52.1	0.12

The polarimetric backscatter measurements given in Table 2 were inserted into equation (7) in order to estimate the effect on measurements made by a radar capable only of transmitting and receiving H-polarized waves. A range of Faraday rotation angles was used between O and 180 degrees. The results are summarized in Figure 1. From the figure, it is clear that a Faraday rotation angle of 40 degrees will lead to a significant drop in the measured backscatter level for all scatterer t ypcs. Further results for L-Band and P-Band data will be presented at the workshop. -

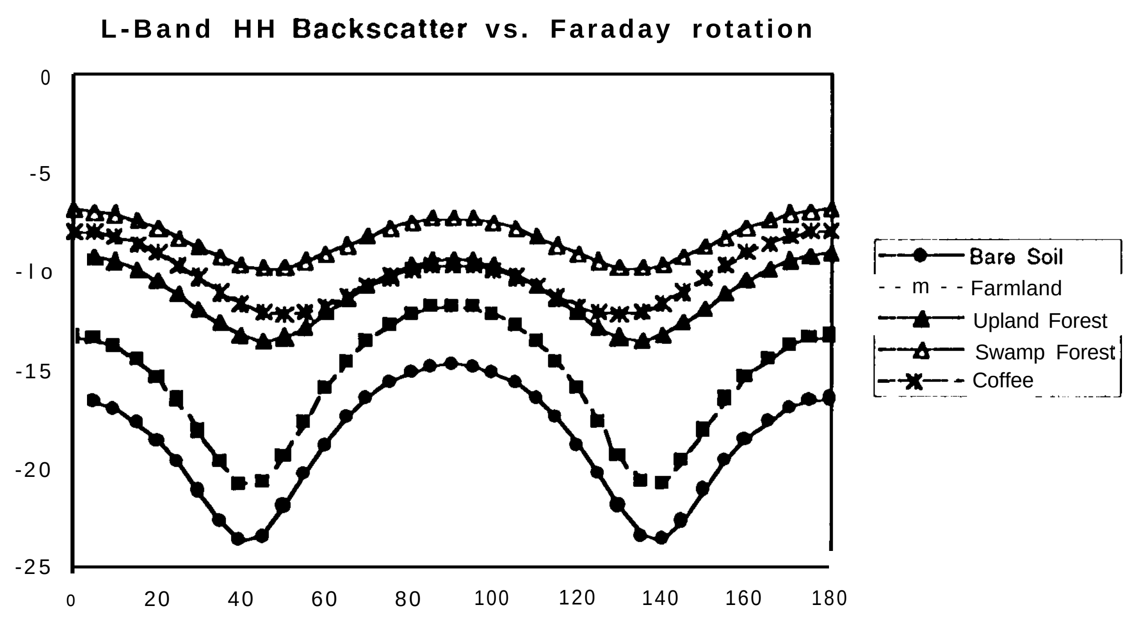

Figure 1: Predicted backscatter values as measured by an L-Band H-transmit, H-receive radar for different backscatter types as a function of Faraday rotation angle, Ω.

4. CORRECTION FOR FARADAY ROTATION

This section addresses the problem of estimating the Faraday rotation Ω, and correcting measured data,

4.1 HH polarization measurements only

From equation (7), which has four unknowns, it is clear that there is no way to estimate Ω from single-pol measurements. Even if Ω were known by other means, it is not possible to correct the HH measurements without knowledge of the VV backscatter.

4.2 Dual-polarized (HH and HV) measurements

Referring to the HH and HV backscatter measurements in (6a) and (6b) we have two equations with 4 unknowns. Forming cross-products as in (11) this becomes 3 equations in 5 unknowns (assuming that like- and cross-polarized scattering matrix terms are uncorrelated). It is clear that, to estimate Ω from dual-polarized measurements, some assumptions about the polarimetric behavior of the scatterers are necessary. Even if such assumptions (including various types of symmetry) are made, preliminary analysis indicates that Ω can only be estimated modulo $\pi/2$, which is not sufficient to correct the dual-polarized measurements.

4.2 Quad-polarized measurements

For LightSAR and other longer wavelength polarimetric spaceborne SARS, it is straightforward to start from equation (6) and estimate the Faraday rotation angle, Ω, via:

$$\Omega = \frac{1}{2}\tan^{-1}\left[\frac{\left(M_{vh} - M_{hv}\right)}{\left(M_{hh} + M_{vv}\right)}\right] \qquad (15)$$

for any type of scatterer. Since speckle and additive noise may be present in the backscatter signatures, Ω may be better estimated from averaged second-order statistics, by first calculating:

$$Z_{hv} = 0.5\left(M_{vh} - M_{hv}\right) \qquad (16)$$

then estimating Ω from:

$$\Omega = \frac{1}{2}\tan^{-1}\sqrt{\frac{\left\langle Z_{hv} Z^{*}_{hv}\right\rangle}{\left(\left\langle M_{hh} M^{*}_{hh}\right\rangle + \left\langle M_{vv} M^{*}_{vv}\right\rangle + 2\,\mathrm{Re}\left\{\left\langle M_{hh} M^{*}_{vv}\right\rangle\right\}\right)}} \qquad (17)$$

To correct for a Faraday rotation of Ω, the following matrix multiplication should suffice:

$$\tilde{\mathbf{S}} = \mathbf{R}^{t}\mathbf{M}\mathbf{R}^{t} \tag{18}$$

where $\mathbf{R}^{t} \equiv \mathbf{R}^{-1}$. Expanding the matrix terms out, (18) can be written:

$$\begin{bmatrix} \tilde{S}_{hh} & \tilde{S}_{vh} \\ \tilde{S}_{hv} & \tilde{S}_{vv} \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega \\ \sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{bmatrix} \begin{bmatrix} \cos\Omega & -\sin\Omega \\ \sin\Omega & \cos\Omega \end{bmatrix} \tag{19}$$

Since values of tan-* are between $\pm\pi/2$, values of Ω estimated using (17) will lie between $\pm\pi/4$. Put another way, this means that Ω can only be estimated modulo n/2 from (17), which is not sufficient. It is straightforward to show that, with an error in Ω of n/2, the estimate for the corrected scattering matrix from (19) will give:

$$\begin{bmatrix} \tilde{S}_{hh} & \tilde{S}_{vh} \\ \tilde{S}_{hv} & \tilde{S}_{vv} \end{bmatrix} = \begin{bmatrix} -S_{vv} & -S_{hv} \\ S_{vh} & -S_{hh} \end{bmatrix} \tag{20}$$

which is clearly in error. Fortunately, this problem may be identified from the cross-pol terms by comparing measurements before correction [from eq. (13)] and after:

$$\left\langle \tilde{S}_{hv}\tilde{S}^{*}_{vh} \right\rangle = -\left\langle M'_{hv}M'^{*}_{hv} \right\rangle$$

and

$$0.25\left\langle \left(\tilde{S}_{hv} + \tilde{S}_{vh}\right)\left(\tilde{S}_{hv} + \tilde{S}_{vh}\right)^{*} \right\rangle \neq \left\langle M'_{hv}M'^{*}_{hv} \right\rangle \tag{21}$$

This test should readily reveal the presence of a n/2 error in Ω, provided that the cross-pol backscatter S_{hv} (or S_{vh}) is not identically zero. This condition can be determined in advance by examining the symmetrized cross-pol measurement represented by (13). If the undesired phase change between the two 'corrected' cross-pol estimates is successfully detected using (21), the procedure to follow should be to add (or subtract) n/2 from Ω (which means that the largest error in Ω is now $\pm\pi$), then apply the correction in equation (19) again. It is straightforward to show that, with an error in Ω of π, the estimate for the new corrected scattering matrix will be:

$$\begin{bmatrix} \tilde{S}_{hh} & \tilde{S}_{vh} \\ \tilde{S}_{hv} & \tilde{S}_{vv} \end{bmatrix} = -1 \begin{bmatrix} S_{hh} & S_{vh} \\ S_{hv} & S_{vv} \end{bmatrix} \tag{22}$$

i.e. all scattering matrix elements will be measured correctly, except for an overall phase error of $\pm\pi$. This is largely irrelevant for polarimetric SAR data analysis, in which the cross-products of (22) are of most interest. A phase error of $\pm\pi$ may be significant for analysis of repeat-pass interferometry data, however, in which relative phases between passes are of most interest.

5. SUMMARY

We have modeled the effects of Faraday rotation on spaceborne SAR backscatter signatures and presented a new approach for estimating Ω and correcting fully polarimetric measurements. This approach improves on the numerical solution offered in (Gail, 1993) and the approach using ground-based corner reflectors described in (Rosen, 1992). The approach described here also allows correction for modulo $\pi/2$ errors in the estimate for Ω, provided the cross-pol backscatter is not zero. It may be possible to correct for a modulo $\pi/2$ error even in this case, since zero cross-pol backscatter is usually associated with (slightly) rough surface scattering, for which the W backscatter term is typically greater than or equal to the HH. Thus an examination of the scattering matrix elements may reveal a switch between the two like-polarized measurements due to a n/2 error in Ω as in (20).

The approach described in this paper allows recovery of the Faraday rotation angle Ω from polarimetric backscatter measurements made by a longer wavelength (e.g. >20 cm) SAR. Further work will focus on whether the estimates for Ω derived from such backscatter measurements can be used to derive estimates for Total Electron Content (TEC) " at high spatial resolution, over large areas.

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